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Research Paper

Unveiling Multivariable Hermite-Based Genocchi Polynomials: Insights from Factorization Method

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Abstract

This paper presents novel polynomials resulting from the convolution of generalized multivariable Hermite polynomials and Genocchi polynomials. Investigating their properties, such as recurrence relations, explicit formulas utilizing shift operators, and differential equations, forms the core of our exploration. Moreover, we derive integro-differential and partial differential equations for these polynomials, thereby enriching the comprehension and applicability of these hybrid polynomials across diverse mathematical domains.

Key Words: Appell Polynomials, Multivariable-Hermite Polynomials, Recurrence Relation, Shift Operators, Differential Equations, Applications
AMS 2020 Classification: 33E20, 33C55, 33B10, 33C45, 34D05, 45D05

1. Introduction and Preliminaries

To elucidate the fundamental principle of the factorization method, we briefly explore the comparison between Maxwell's and Dirac's equations, which are both pivotal in physics. Both systems exhibit similar traits, including the utilization of first-order partial derivatives and adherence to Lorentz invariance principles. However, a crucial distinction arises from the linearity inherent in Maxwell's equations, which can lead to challenges associated with infinite self-energies. In simpler terms, although these systems share mathematical characteristics and Lorentz invariance, the issue of infinite self-energies is uniquely pertinent to Maxwell's equations due to their linear nature.

The factorization method, extensively employed in physics to tackle eigenvalue problems, involves solving two first-order differential equations that, when combined, yield a second-order differential equation of equal significance. Additionally, this method encompasses the computation of transition probabilities, considering the production process, and provides a comprehensive framework for effectively addressing perturbation issues. In essence, it leverages the solution of two specific types of differential equations to obtain another equally significant equation, incorporating transition probabilities to understand how a system evolves. Moreover, it offers a flexible approach to handle perturbation concerns and disruptions that may affect system stability or accuracy.

We consider a sequence of polynomials denoted as $\mathcal{P}_n(j_1)_{n=0}^{\infty}$, where n signifies the polynomial degree. Within this context, two sequences of differential operators, $\mathfrak{G}n^-$ and $\mathfrak{G}n^+$, act upon this polynomial sequence. These operators exhibit specific properties:

$$\mathcal{P}_{n-1}(j_1) = \mathfrak{G}n^{-}(\mathcal{P}_n(j_1))$$

and

$$\mathcal{P}_{n+1}(j_1) = \mathfrak{G}n^{+}(\mathcal{P}_n(j_1)).$$

An important property, known as the differential equation, is expressed as follows:

$$\mathcal{P}_n(j_1) = (\mathfrak{G}n + 1^{-}\mathfrak{G}n^{+})\{\mathcal{P}_n(j_1)\}. \tag{1}$$

This equation serves as a foundational element for constructing differential equations using the factorization method. The primary objective is to identify two distinct operators: the multiplicative operator $\mathfrak{G}n^{+}$ and the derivative operator $\mathfrak{G}n^{-}$, ensuring they fulfill the conditions laid out in equation (1). By methodically deriving operators that adhere to this equation, we can systematically construct differential equations through the factorization approach, offering new perspectives and simplifying the analysis and solution of the equation.

The operational rule guides specific operations or computations associated with these polynomials. It elucidates how to modify or evaluate the 2VHKdFP using particular mathematical operations or transformations.

These polynomials are defined by the generating expression:

$$\sum_{n=0}^{\infty} \mathcal{Y}_n^{[2]}(j_1, j_2)\frac{\xi^n}{n!} = \exp(j_1\xi + j_2\xi^2) \tag{2}$$

and represented by the series:

$$\mathcal{Y}_n^{[2]}(j_1, j_2) = n! \sum_{k=0}^{[\frac{n}{2}]} \frac{j_2^k \, j_1^{n-2k}}{k!(n-2k)!}.$$

Moreover, the Polynomials $\mathcal{Y}_n^{[m]}(j_1, j_2, \cdots, j_m)$, referred to as multivariable Hermite Polynomials (MHP), are expressed by the relation:

$$\exp(j_1\xi + j_2\xi^2 + \cdots + j_m\xi^m) = \sum_{n=0}^{\infty} \mathcal{Y}_n^{[m]}(j_1, j_2, \cdots, j_m)\frac{\xi^n}{n!}.$$

With the operational rule:

$$\exp\left(j_2\frac{\partial^2}{\partial {j_1}^2} + j_3\frac{\partial^3}{\partial {j_1}^3} + \cdots + j_m\frac{\partial^m}{\partial {j_1}^m}\right)j_1^n = \mathcal{Y}_n^{[m]}(j_1, j_2, \cdots, j_m), \tag{3}$$

and the series representation:

$$\mathcal{Y}_n^{[m]}(j_1, j_2, \cdots j_m) = n! \sum_{r=0}^{[n/m]} \frac{j_m^r \mathcal{Y}_{n-mr}^{[m]}(j_1, j_2, \cdots, j_{m-1})}{r! \, (n-mr)!}.$$

Special functions, equations, and integers constitute essential subjects of study across numerous branches of mathematics, physics, and engineering. Among these, Genocchi polynomials and numerals hold significant importance, frequently appearing in fundamental and applied mathematical contexts related to approximation theories, interpolation problems, and quadrature rules, as noted in [1, 2, 3, 4, 5]. Several authors have extensively investigated various extensions of Genocchi polynomials, contributing to the development of mathematical theory and its applications. Notable works include [6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The Genocchi polynomials of order r, denoted as $\mathfrak{F}_n^{[r]}(j_1)$, as defined by:

$$\sum_{n=0}^{\infty} \mathfrak{F}_n^{[r]}(j_1)\, \frac{\xi^n}{n!} = \left(\frac{2\xi}{e^{\xi}+1}\right)^r \, e^{j_1\xi}, \tag{4}$$

where $n \in \mathbb{Z}^{+}$.

Setting $j_1 = 0$ in equation (4) yields the corresponding Genocchi numbers $\mathfrak{F}_n^{[r]}$ of order r:

$$\mathfrak{F}_n^{[r]} := \mathfrak{F}_n^{[r]}(0)$$

The factorization method, pioneered by He and Ricci [16], has found widespread application in deriving differential equations for various types of polynomials, including Appell polynomials and their multivariable extensions, Bernoulli and Euler polynomials, as detailed in [17] and [18]. Moreover, this technique has been extended to derive expressions such as integrodifferential and partial differential equations for hybrid forms, 2D extended, and mixed-type Appell family polynomials, as evidenced in [15] and [19]. Ozarslan [20] utilized the Appell polynomials to generate a series of finite order differential equations by expanding the factorization technique using k-times shift operators. These findings have significantly contributed to establishing recurrence relations, shift operators, and families of differential equations for multivariable Hermite Appell Polynomials, as indicated by equation (5) in this paper. Undoubtedly, the factorization approach serves as an invaluable tool for deriving equations for diverse types of polynomials across various mathematical domains.

The advancement of operational techniques, recurrence relations, shift operators, and families of differential equations for various polynomial types in different mathematical domains has spurred the development of multivariable Hermite-Genocchi polynomials. Denoted by ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$, these polynomials are generated by the expression:

$$\frac{2\xi}{e^{\xi}+1} \exp\left(j_1\xi + j_2\xi^2 + \cdots + j_m\xi^m = \sum_{n=0}^{\infty} {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!}. \tag{5}$$

This generating expression involves applying a suitable linear operator on the product of m Hermite polynomials.

The subsequent sections of this article delve into exploring the properties and characteristics of multivariable Hermite-based Genocchi polynomials. Section 2 elucidates the generating relation, recurrence relation, and shift operators for these polynomials. Furthermore, Section 3 delves into the development of several families of differential equations tailored to these polynomials. Finally, the concluding section offers a summary of the findings presented in this article.

2. Recurrence Relations and Shift Operators

This section aims to derive shift operators and recurrence relations for the MVHGP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$. Throughout this derivation process, relationships between different instances of the MVHGP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ with varying values of the indices n, j_1, j_2, j_3, and so forth are established. These recurrence relations enable the representation of polynomials in terms of one another, facilitating faster computations and identification of recurrent patterns. By establishing these relations and shift operators, a deeper understanding of the properties and behaviors of the MVHGP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ is achieved, which can prove beneficial in various computational, analytical, or application contexts involving these polynomials. The recurrence relation for the function ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ is derived based on the following outcome:

Theorem 1. *The MVHGP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ fulfill the following recurrence relation:*

$$\begin{aligned}
{}_{\mathcal{Y}}\mathfrak{G}_{n+1}^{[m]}(j_1, j_2, \cdots, j_m) &= (j_1 - \frac{1}{2})\ {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m) + \frac{1}{2}\sum_{k=1}^{n}\binom{n}{k}\mathfrak{G}_k\,{}_{\mathcal{Y}}\mathfrak{G}_{n-k+1}^{[m]}(j_1, j_2, \cdots, j_m) \\
&+ j_2\,{}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1, j_2, \cdots, j_m) + 3n(n-1)j_3\,{}_{\mathcal{Y}}\mathfrak{G}_{n-2}^{[m]}(j_1, j_2, \cdots, j_m) \\
&+ \cdots + n(n-1)(n-2)\cdots(n-m+1)j_m\,{}_{\mathcal{Y}}\mathfrak{G}_{n-m}^{[m]}(j_1, j_2, \cdots, j_m), \qquad (6)
\end{aligned}$$

where $\mathfrak{G}_k$ denotes the Genocchi numbers of order k.

Proof. Taking the derivatives of (5) w.r.t. ξ, we find

$$\begin{aligned}\sum_{n=0}^{\infty} {}_{\mathcal{Y}}\mathcal{G}_{n+1}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} &= \left(j_1 + 2\ j_2 t + 3\ j_3 t^2 + \cdots + m\ j_m t^{m-1}\right)\sum_{n=0}^{\infty} {}_{\mathcal{Y}}\mathcal{G}_n(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &- \frac{1}{2}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} {}_{\mathcal{Y}}\mathcal{G}_n(j_1, j_2, j_3, \cdots, j_m)\mathfrak{G}_k\frac{\xi^{n+k}}{n!\ k!}.\end{aligned}$$

The Cauchy-product rule is applied to the right-hand side (RHS) after simplification and we arrive at the following conclusion:

$$\begin{aligned}\sum_{n=0}^{\infty} {}_{\mathcal{Y}}\mathcal{G}_{n+1}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} &= \sum_{n=0}^{\infty} j_1\ {}_{\mathcal{G}}{}_n(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} + \sum_{n=0}^{\infty} 2n\ j_2\ {}_{\mathcal{Y}}\mathcal{G}_{n-1}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &+ \sum_{n=0}^{\infty} 3n(n-1)j_3\ {}_{\mathcal{Y}}\mathcal{G}_{n-2}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &+ \cdots + \sum_{n=0}^{\infty} n(n-1)\cdots(n-m+1)\ m\ j_m {}_{\mathcal{Y}}\mathcal{G}_{n-m}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &- \frac{1}{2}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k} {}_{\mathcal{Y}}\mathcal{G}_{n-k}(j_1, j_2, j_3, \cdots, j_m)\mathfrak{G}_k\frac{\xi^n}{n!}.\end{aligned}$$

Further simplifying previous expression , it follows that

$$\begin{aligned}\sum_{n=0}^{\infty} {}_{\mathcal{Y}}\mathcal{G}_{n+1}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} &= \sum_{n=0}^{\infty}\left(j_1 - \frac{1}{2}\right){}_{\mathcal{G}}{}_n(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} + \sum_{n=0}^{\infty} 2n\ j_2\ {}_{\mathcal{Y}}\mathcal{G}_{n-1}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &+ \sum_{n=0}^{\infty} 3n(n-1)j_3\ {}_{\mathcal{Y}}\mathcal{G}_{n-2}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &+ \cdots + \sum_{n=0}^{\infty} n(n-1)\cdots(n-m+1)\ m j_m {}_{\mathcal{Y}}\mathcal{G}_{n-m}(j_1, j_2, j_3, \cdots, j_m)\frac{\xi^n}{n!} \\ &+ \frac{1}{2}\sum_{n=0}^{\infty}\sum_{k=1}^{n}\binom{n}{k} {}_{\mathcal{Y}}\mathcal{G}_{n-k+1}(j_1, j_2, j_3, \cdots, j_m)\mathfrak{G}_k\frac{\xi^n}{n!}.\end{aligned}$$

Comparing the coefficients of like powers of ξ on both sides of previous expression, assertion (6) is established □

In the following analysis, we present the construction of shift operators for the MVHGP $\mathcal{Y}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)$ by establishing the subsequent result:

Theorem 2. *The MVHGP* ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)$ *satisfy the listed shift operators:*

$$ {}_{j_1}\pounds_n^- := \frac{1}{n}D_{j_1}, \tag{7}$$

$$ {}_{j_2}\pounds_n^- := \frac{1}{n}D_{j_1}^{-1}D_{j_2}, \tag{8}$$

$$ {}_{j_3}\pounds_n^- := \frac{1}{n}D_{j_1}^{-2}D_{j_3}, \tag{9}$$

$$\vdots$$

$$ {}_{j_m}\pounds_n^- := \frac{1}{n}D_{j_1}^{-(m-1)}D_{j_m}, \tag{10}$$

$$ {}_{j_1}\mathcal{L}_n^+ := (j_1 - \frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^k + 2j_2 D_{j_1} + 3j_3 D_{j_1}^2 + \cdots + m j_m D_{j_1}^{m-1} \tag{11} $$

$$ {}_{j_2}\mathcal{L}_n^+ := (j_1 - \frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{k-1} + 2j_2\ D_{j_1}^{-1} D_{j_2} + 3j_3\ D_{j_1}^{-2} D_{j_2}^2 + \cdots + m j_m\ D_{j_1}^{-(m-1)} D_{j_2}^{m-1}, \tag{12} $$

$$ {}_{j_3}\mathcal{L}_n^+ := (j_1 - \frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{k-1} + 2j_2 D_{j_1}^{-2} D_{j_3} + 3j_3\ D_{j_1}^{-4} D_{j_3}^2 + \cdots + m j_m\ D_{j_1}^{-2(m-1)} D_{j_3}^{m-1}, \tag{13} $$

$$ \vdots $$

$$ \begin{aligned} {}_{j_m}\mathcal{L}_n^+ &:= (j_1 - \frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)^2} D_{j_m}^{k-1} + 2j_2 D_{j_1}^{-(m-1)} D_{j_m} \\ &+ 3j_3\ D_{j_1}^{-2(m-1)} D_{j_m}^2 + \cdots + m j_m\ D_{j_1}^{-(m-1)^2} D_{j_m}^{m-1}, \end{aligned} \tag{14} $$

where

$$ D_{j_1} := \frac{\partial}{\partial j_1}, \quad D_{j_2} := \frac{\partial}{\partial j_2}, \quad D_{j_3} := \frac{\partial}{\partial j_3} \quad \textit{and} \quad D_{j_1}^{-1} := \int_0^{j_1} f(\eta) d\eta. $$

Proof. By applying differentiation to equation (5) with respect to j_1 and then equating coefficients of similar powers of ξ on both sides, we derive the following equation:

$$ \frac{\partial}{\partial j_1}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = n\ {}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1, j_2, \cdots, j_m), \tag{15} $$

As a consequence of the aforementioned steps, we arrive at the following expression:

$$ {}_{j_1}\mathcal{L}_n^- \{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = \frac{1}{n} D_{j_1}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1, j_2, \cdots, j_m), \tag{16} $$

thus proving the validity of assertion (7).

Upon differentiating equation (5) with respect to j_2 and equating the coefficients of the corresponding powers of ξ on both sides, we arrive at the following expression:

$$ \frac{\partial}{\partial j_2}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = n(n-1)\ {}_{\mathcal{Y}}\mathfrak{G}_{n-2}^{[m]}(j_1, j_2, \cdots, j_m). $$

The previous expression can alternatively be expressed as

$$ \frac{\partial}{\partial j_2}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = n\ \frac{\partial}{\partial j_1}\{{}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1, j_2, \cdots, j_m)\}, $$

thus eventually gives

$$ {}_{j_2}\mathcal{L}_n^- \{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = \frac{1}{n} D_{j_1}^{-1} D_{j_2}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1, j_2, \cdots, j_m). \tag{17} $$

Therefore the validity of assertion (8) is established.

After differentiating equation (5) with respect to j_3 and equating the coefficients of the identical powers of ξ on both sides, we obtain the following derived expression:

$$\frac{\partial}{\partial j_3}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = n(n-1)(n-2)\ {}_{\mathcal{Y}}\mathfrak{G}_{n-3}^{[m]}(j_1,j_2,\cdots,j_m). \tag{18}$$

The previous equation (18) can also be expressed as

$$\frac{\partial}{\partial j_3}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = n\ \frac{\partial^2}{\partial j_1^2}\{{}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1,j_2,\cdots,j_m)\},$$

thus eventually gives

$${}_{j_3}\pounds_n^-\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = \frac{1}{n}D_{j_1}^{-2}D_{j_3}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1,j_2,\cdots,j_m). \tag{19}$$

Hence, yielding assertion (9).

Finally, upon differentiating equation (5) with respect to j_m and equating the coefficients of the same powers of ξ on both sides of the resulting equation, we arrive at the following expression:

$$\frac{\partial}{\partial j_m}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = n(n-1)(n-2)(n-m+1){}_{\mathcal{Y}}\mathfrak{G}_{n-m}^{[m]}(j_1,j_2,\cdots,j_m),$$

and further presented as

$$\frac{\partial}{\partial j_m}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = n\ \frac{\partial^{m-1}}{\partial j_1^{m-1}}\{{}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1,j_2,\cdots,j_m)\}$$

and finally gives

$${}_{j_m}\pounds_n^-\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = \frac{1}{n}D_{j_1}^{-(m-1)}D_{j_m}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_{n-1}^{[m]}(j_1,j_2,\cdots,j_m). \tag{20}$$

Therefore, the validity of assertion (10) is established.

To establish the equation for the raising operator (11), we employ the following expression:

$${}_{\mathcal{Y}}\mathfrak{G}_{n-k}^{[m]}(j_1,j_2,\cdots,j_m) = ({}_{j_1}\pounds_{n-k+1}^-\ {}_{j_1}\pounds_{n-k+2}^-\cdots{}_{j_1}\pounds_{n-1}^-\ {}_{j_1}\pounds_n^-)\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\}, \tag{21}$$

Thus, in view of expression (16), expression (21) in simplified form can be presented as:

$${}_{\mathcal{Y}}\mathfrak{G}_{n-k}^{[m]}(j_1,j_2,\cdots,j_m) = \frac{(n-k)!}{n!}D_{j_1}^k\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m)\}. \tag{22}$$

By substituting equation (22) into the recurrence relation (6), we deduce that:

$$\begin{aligned}{}_{\mathcal{Y}}\mathfrak{G}_{n+1}^{[m]}(j_1,j_2,\cdots,j_m) \quad &= \quad \left((j_1-\frac{1}{2})+\sum_{k=1}^{n}\frac{\mathfrak{G}_k}{k!}D_{j_1}^k+2j_2D_{j_1}+3j_3D_{j_1}^2+\cdots+mj_m\ D_{j_1}^{m-1}\right)\\ &\times \quad {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1,j_2,\cdots,j_m).\end{aligned}$$

Thus (11) of raising operator ${}_{j_1}\pounds_n^+$ is proved.

In order to demonstrate the raising operator (12), we examine the following relation:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = ({}_{j_2}\mathcal{L}^-_{n-k+1}\ {}_{j_2}\mathcal{L}^-_{n-k+2}\cdots {}_{j_2}\mathcal{L}^-_{n-1}\ {}_{j_2}\mathcal{L}^-_{n})\{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}, $$

By taking into account equation (17), we can expand the above expression as follows:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = \frac{(n-k)!}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{(k-1)} \{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}. \tag{23} $$

By substituting equation (23) into the recurrence relation (6), we can deduce that:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n+1}(j_1,j_2,\cdots,j_m) = \Bigg((j_1-\frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{k-1} + 2j_2 D_{j_1}^{-1} D_{j_2} + 3j_3 D_{j_1}^{-2} D_{j_2}^{2} + \cdots + m j_m D_{j_1}^{-(m-1)} D_{j_2}^{m-1} \Bigg). $$

Therefore, we have successfully established the validity of Assertion (12) for the raising operator ${}_{j_2}\mathcal{L}^+_n$.

To demonstrate the raising operator $j_3\mathcal{L}^+ n$, we consider the following expression:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = ({}_{j_3}\mathcal{L}^-_{n-k+1}\ {}_{j_3}\mathcal{L}^-_{n-k+2}\cdots {}_{j_3}\mathcal{L}^-_{n-1}\ {}_{j_3}\mathcal{L}^-_{n})\{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}. $$

By taking into account equation (19), we can expand the above expression as follows:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = \frac{(n-k)!}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{(k-1)} \{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}. \tag{24} $$

By substituting equation (24) into the recurrence relation (6), we find that:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n+1}(j_1,j_2,\cdots,j_m) = \Bigg((j_1-\frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{k-1} + 2j_2 D_{j_1}^{-2} D_{j_3} + 3j_3 D_{j_1}^{-4} D_{j_3}^{2} + \cdots + m j_m D_{j_1}^{-2(m-1)} D_{j_3}^{m-1} \Bigg). $$

Therefore, we have successfully established the validity of Assertion (13) for the raising operator ${}_{j_3}\mathcal{L}^+_n$.

Finally, To establish the raising operator ${}_{j_m}\mathcal{L}^+_n$, we analyze the following expression:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = ({}_{j_m}\mathcal{L}^-_{n-k+1}\ {}_{j_m}\mathcal{L}^-_{n-k+2}\cdots {}_{j_m}\mathcal{L}^-_{n-1}\ {}_{j_m}\mathcal{L}^-_{n})\{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}. $$

By taking into account equation (20), we can expand the above expression as follows:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n-k}(j_1,j_2,\cdots,j_m) = \frac{(n-k)!}{k!} D_{j_1}^{-(k-1)^2} D_{j_m}^{(k-1)} \{{}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n}(j_1,j_2,\cdots,j_m)\}. \tag{25} $$

By substituting equation (25) into the recurrence relation (6), we deduce that:

$$ {}_{\mathcal{Y}}\mathfrak{G}^{[m]}_{n+1}(j_1,j_2,\cdots,j_m) = \Bigg((j_1-\frac{1}{2}) + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_m}^{k-1} + 2j_2 D_{j_1}^{-(m-1)} D_{j_m} + 3j_3 D_{j_1}^{-2(m-1)} D_{j_m}^{2} + \cdots + m j_m D_{j_1}^{-(m-1)^2} D_{j_m}^{m-1} \Bigg). $$

Thus, expression (14) of raising operator ${}_{j_m}\mathcal{L}^+_n$ is proved.
□

In the following section, we undertake an examination of the families of differential equations satisfied by the multivariable Hermite-based Genocchi polynomials. This section offers a comprehensive exploration of various categories of differential equations, including ordinary differential equations, integro-differential equations, and partial differential equations. These equations are derived using the factorization method.

3. Analysis of Differential Equation Families

In this section, we provide detailed explanations of each type of equation, elucidating its structure and its connection to the multivariable Hermite-based Genocchi polynomials. For the MVHGP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)$, we establish the differential, integro-differential, and partial differential equations. Furthermore, we derive the differential equation for the M*D*HAP ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, \cdots, j_m)$ by presenting the following conclusion:

Theorem 3. *The MVHGP* ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ *satisfy the following differential equation:*

$$\left((j_1 - \frac{1}{2})D_{j_1} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{k+1} + 2j_2 D_{j_1}^2 + 3j_2 D_{j_1}^3 + \cdots + j_m D_{j_1}^m - n\right) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0.$$

Proof. The expressions (7) and (11) of the shift operators are utilized in the factorization relation, as follows:

$${}_{j_1}\mathcal{L}_{n+1}^{-}\ {}_{j_1}\mathcal{L}_n^{+}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m).$$

Simplifying the mathematical expression, assertion (**??**) is proved. □

Theorem 4. *The MVHGP* ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ *satisfy the following integro-differential equations:*

$$\left((j_1 - \frac{1}{2})D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{k} + 2j_2\ D_{j_1}^{-1} D_{j_2}^2 + 3j_3\ D_{j_1}^{-2} D_{j_2}^3 + \cdots \right.$$
$$\left. + mj_m\ D_{j_1}^{-(m-1)} D_{j_2}^{m} - (n+1)D_{j_1}\right) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{26}$$

$$\left((j_1 - \frac{1}{2})D_{j_3} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{k-1} D_{j_3} + 2j_2\ D_{j_1}^{-1} D_{j_2} D_{j_3} + 3j_3\ D_{j_1}^{-2} D_{j_2}^2 D_{j_3} + \cdots \right.$$
$$\left. + mj_m\ D_{j_1}^{-(m-1)} D_{j_2}^{m-1} D_{j_3} - (n+1)D_{j_1}^2\right) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{27}$$

$$\left((j_1 - \frac{1}{2})D_{j_m} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)} D_{j_2}^{k-1} D_{j_m} + 2j_2\ D_{j_1}^{-1} D_{j_2} D_{j_m} + 3j_3\ D_{j_1}^{-2} D_{j_2}^2 D_{j_m} + \cdots \right.$$
$$\left. + mj_m\ D_{j_1}^{-(m-1)} D_{j_3}^{m-1} D_{j_m} - (n+1)D_{j_1}^{m-1}\right) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{28}$$

$$\left((j_1 - \frac{1}{2})D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{k-1} D_{j_2} + 2j_2 D_{j_1}^{-2} D_{j_3} D_{j_2} + 3j_3\ D_{j_1}^{-4} D_{j_3}^2 D_{j_2} + \cdots \right.$$
$$\left. + mj_m\ D_{j_1}^{-2(m-1)} D_{j_3}^{m-1} D_{j_2} - (n+1)D_{j_1}\right) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{29}$$

$$\left((j_1 - \frac{1}{2})D_{j_3} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{k} + 2j_2 D_{j_1}^{-2} D_{j_3}^2 + 3j_3\ D_{j_1}^{-4} D_{j_3}^3 + \cdots \right.$$

$$+mj_m\ D_{j_1}^{-2(m-1)} D_{j_3}^{m} - (n+1) D_{j_1}^{2} \Big) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{30}$$

$$\Bigg((j_1 - \frac{1}{2}) D_{j_m} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-2(k-1)} D_{j_3}^{k-1} D_{j_m} + 2j_2 D_{j_1}^{-2} D_{j_3} D_{j_m} + 3j_3\ D_{j_1}^{-4} D_{j_3}^{2} D_{j_m} + \cdots$$

$$+mj_m\ D_{j_1}^{-2(m-1)} D_{j_3}^{m-1} D_{j_m} - (n+1) D_{j_1}^{m-1} \Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{31}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\Bigg((j_1 - \frac{1}{2}) D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)^2} D_{j_m}^{k-1} D_{j_2} + 2j_2 D_{j_1}^{-(m-1)} D_{j_m} D_{j_2} + 3j_3\ D_{j_1}^{-2(m-1)} D_{j_m}^{2} D_{j_2} + \cdots$$

$$+mj_m\ D_{j_1}^{-(m-1)^2} D_{j_m}^{m-1} D_{j_2} - (n+1) D_{j_1} \Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{32}$$

$$\Bigg((j_1 - \frac{1}{2}) D_{j_3} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)^2} D_{j_m}^{k-1} D_{j_3} + 2j_2 D_{j_1}^{-(m-1)} D_{j_m} D_{j_3} + 3j_3\ D_{j_1}^{-2(m-1)} D_{j_m}^{2} D_{j_3} + \cdots$$

$$+mj_m\ D_{j_1}^{-(m-1)^2} D_{j_m}^{m-1} D_{j_3} - (n+1) D_{j_1}^{2} \Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{33}$$

$$\Bigg((j_1 - \frac{1}{2}) D_{j_m} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{-(k-1)^2} D_{j_m}^{k} + 2j_2 D_{j_1}^{-(m-1)} D_{j_m}^{2} + 3j_3\ D_{j_1}^{-2(m-1)} D_{j_m}^{3} + \cdots$$

$$+mj_m\ D_{j_1}^{-(m-1)^2} D_{j_m}^{m} - (n+1) D_{j_1}^{m-1} \Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \tag{34}$$

Proof. Utilizing the expression

$$\pounds_{n+1}^{-}\ \pounds_n^{+}\{{}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)\} = {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m). \tag{35}$$

By substituting expressions (8) and (12) into the factorization relation (35), we establish the validity of Assertion (26).

By utilizing the expressions (9) and (12) in the factorization relation (34), we establish the validity of Assertion (27).

By employing the expressions (10) and (12) in the factorization relation (34), we verify the validity of Assertion (28).

Using expressions (8), (9), and (10) along with expression (13), we can separately prove assertions (29), (30), and (31).

By utilizing expressions (8), (9), and (10) in conjunction with expression (14), we can independently demonstrate the validity of assertions (32), (33), and (34). □

Theorem 5. *The MVHGP* ${}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m)$ *satisfy the following partial differential equations:*

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^n D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{n-k+1} D_{j_2}^k + 2j_2\ D_{j_1}^{n-1} D_{j_2}^2 + 3j_3\ D_{j_1}^{n-2} D_{j_2}^3 + \cdots$$
$$+ m j_m\ D_{j_1}^{n-(m-1)} D_{j_2}^m - (n+1) D_{j_1}^{n+1}\Bigg) \times {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \quad (36)$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^n D_{j_3} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{n-k+1} D_{j_2}^{k-1} D_{j_3} + 2j_2\ D_{j_1}^{n-1} D_{j_2} D_{j_3} + 3j_3\ D_{j_1}^{n-2} D_{j_2}^2 D_{j_3} + \cdots$$
$$+ m j_m\ D_{j_1}^{n-(m-1)} D_{j_2}^{m-1} D_{j_3} - (n+1) D_{j_1}^{n+2}\Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \quad (37)$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^{2n} D_{j_m} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{2n-k+1} D_{j_2}^{k-1} D_{j_m} + 2j_2\ D_{j_1}^{2n-1} D_{j_2} D_{j_m} + 3j_3\ D_{j_1}^{2n-2} D_{j_2}^2 D_{j_m} + \cdots$$
$$+ m j_m\ D_{j_1}^{2n-(m-1)} D_{j_3}^{m-1} D_{j_m} - (n+1) D_{j_1}^{2n+m-1}\Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \quad (38)$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^{2n+2} D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{2n-2k+2} D_{j_3}^{k-1} D_{j_2} + 2j_2 D_{j_1}^{2n} D_{j_3} D_{j_2} + 3j_3\ D_{j_1}^{2n-2} D_{j_3}^2 D_{j_2} + \cdots$$
$$+ m j_m\ D_{j_1}^{2n-2m} D_{j_3}^{m-1} D_{j_2} - (n+1) D^{2n+m-1} j_1\Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \quad (39)$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^{2n+2} D_{j_3} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{2n-2k+2} D_{j_3}^{k} + 2j_2 D_{j_1}^{2n} D_{j_3}^2 + 3j_3\ D_{j_1}^{2n-2} D_{j_3}^3 + \cdots$$
$$+ m j_m\ D_{j_1}^{2n-2m} D_{j_3}^{m} - (n+1) D_{j_1}^{2n+4}\Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0.$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^{2n+2} D_{j_m} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{2n-2k+2} D_{j_3}^{k-1} D_{j_m} + 2j_2 D_{j_1}^{2n} D_{j_3} D_{j_m} + 3j_3\ D_{j_1}^{2n-2} D_{j_3}^2 D_{j_m} + \cdots$$
$$+ m j_m\ D_{j_1}^{2n-2m} D_{j_3}^{m-1} D_{j_m} - (n+1) D_{j_1}^{2n+m+1}\Bigg) {}_{\mathcal{Y}}\mathfrak{G}_n^{[m]}(j_1, j_2, j_3, \cdots, j_m) = 0. \quad (40)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\Bigg((j_1 - \frac{1}{2})D_{j_1}^{n^2+1} D_{j_2} + \sum_{k=1}^{n} \frac{\mathfrak{G}_k}{k!} D_{j_1}^{n^2-k^2+2k} D_{j_m}^{k-1} D_{j_2} + 2j_2 D_{j_1}^{n^2-m} D_{j_m} D_{j_2} + 3j_3\ D_{j_1}^{n^2-2m-1} D_{j_m}^2 D_{j_2} + \cdots$$

$$+mj_m\ D_{j_1}^{n^2-m^2+2m}\ D_{j_m}^{m-1}D_{j_2}-(n+1)D_{j_1}^{n^2+2}\Big)\mathcal{Y}\mathfrak{G}_n^{[m]}(j_1,j_2,j_3,\cdots,j_m)=0. \tag{41}$$

$$\Big((j_1-\frac{1}{2})D_{j_1}^{n^2+1}D_{j_3}+\sum_{k=1}^{n}\frac{\mathfrak{G}_k}{k!}D_{j_1}^{n^2-k^2+2k}D_{j_m}^{k-1}D_{j_3}+2j_2D_{j_1}^{2n-m}D_{j_m}D_{j_3}+3j_3\ D_{j_1}^{2n-2m}D_{j_m}^2D_{j_3}+\cdots$$

$$+mj_m\ D_{j_1}^{n^2-m^2+2m}\ D_{j_m}^{m-1}D_{j_3}-(n+1)D_{j_1}^{n^2+2}\Big)\mathcal{Y}\mathfrak{G}_n^{[m]}(j_1,j_2,j_3,\cdots,j_m)=0. \tag{42}$$

$$\Big((j_1-\frac{1}{2})D_{j_1}^{n^2+2}D_{j_m}+\sum_{k=1}^{n}\frac{\mathfrak{G}_k}{k!}D_{j_1}^{n^2-k^2+2k}D_{j_m}^{k}+2j_2D_{j_1}^{n^2-m}D_{j_m}^2+3j_3\ D_{j_1}^{n^2-2(m-1)}D_{j_m}^3+\cdots$$

$$+mj_m\ D_{j_1}^{n^2-m^2+2m}D_{j_m}^{m}-(n+1)D_{j_1}^{n^2+m+1}\Big)\mathcal{Y}\mathfrak{G}_n^{[m]}(j_1,j_2,j_3,\cdots,j_m)=0. \tag{43}$$

Proof. Taking derivatives of integro-differential expressions (26) and (27) partially n times w.r.t. j_1, assertion (36) and (37) are proved.

Also, Taking derivatives of integrodifferential expressions (28) partially $2n$ times w.r.t. j_1, assertion (38) is proved.

Further, taking derivatives of integro-differential expressions (29) - (31) partially $2n+2$ times w.r.t. j_1, assertion (39) - (40) are proved.

Furthermore, taking derivatives of integro-differential expressions (32) and (33) partially n^2+1 times w.r.t. j_1, assertion (41) and (42) are proved.

Again, taking derivatives of integrodifferential expressions (34) partially n^2+2 times w.r.t. j_1, assertion (43) is proved. □

4. Conclusion

This study introduces a novel family of hybrid multivariable polynomials obtained by convolving Hermite and Genocchi polynomials, and thoroughly investigates their properties. Specifically, we derive a recurrence relation and a sequence of shift operators satisfied by these multivariable Hermite-Genocchi polynomials. Furthermore, we establish that these polynomials satisfy a differential equation as well as a sequence of integro-differential and partial differential equations. Overall, by proposing this new family of polynomials and examining their characteristics, this paper contributes to the field of polynomial theory.

Moreover, future research and observations could lead to the exploration of new characteristics of these polynomials. This could involve the development of extended and generalized forms, symmetric identities, and the utilization of fractional operators. However, challenges may arise when dealing with determinant forms and summation equations with new datasets.

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