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Research Paper

Fractional k-Calculus Approach to the Extended k-Type Hypergeometric Function

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Abstract

The primary objective of the present manuscript is to evaluate the left-sided and right-sided k-Saigo fractional differentiation and integration of the extended k-hypergeometric function. The study employs Saigo k-type fractional operators, incorporating the k-hypergeometric function within the kernel, to the extended k-hypergeometric function. Additionally, the paper explores special cases associated with k-Riemann-Liouville fractional calculus operators.

Key Words: Extended k-Hypergeometric Function, k-Beta Function; Saigo k-Fractional Calculus

AMS 2020 Classification: 26A33, 33C05, 33B15

1. Introduction

Non-integer order calculus has its origins dating back to 1695. However, it is only in the last twenty years that authors have effectively utilized it, largely due to the advancements in computational resources. The k-fractional calculus represents an extension of classical fractional calculus. Recently, several k-fractional calculus operators have become increasingly popular. The theory of k-fractional calculus operators has been applied to various investigations in recent years. The Saigo k-fractional calculus operator is among the most extensively studied and offers an enhanced formula for fractional derivatives and integration. For example, Gupta and Parihar [1] explored the k-Saigo fractional derivative and integration operators that involve the k-hypergeometric function in the kernel, applying it to the generalized k-Mittag-Leffler function.

In several areas of mathematics and mathematical physics, special functions and their generalized forms are used for finding results of the partial differential equations and fractional differential equations. In 2024, Laxmi et al. [2] introduced a new extension of the k-hypergeometric function and discussed several key properties and results. This function is represented as a power series for $k > 0$, $\Re(\wp_3) > \Re(\wp_2) > s$, $\Re(z_1) > 0$, $\Re(z_2) > 0$, and for all $\frac{1}{k} > |\chi|$.

$$\mathbb{F}_{k,z}^{(z_1,z_2)}\left\{(\wp_1,k),(\wp_2,k);(\wp_3,k);\chi\right\} = \sum_{i=0}^{\infty} \frac{(\wp_1)_{i,k}(\wp_2)_{i,k}}{(\wp_2-sk)_{i,k}} \frac{\mathscr{B}_{k,z}^{(z_1,z_2)}(\wp_2-sk+ik,\wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\wp_3-\wp_2+sk)} \frac{\chi^i}{i!}, \quad (1)$$

where $\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_1,\ \wp_2)$ is an extended k-beta function defined as follows [3]:

$$\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_1,\ \wp_2) = \frac{1}{k}\int_0^1 \tau^{\frac{\wp_1}{k}-1}(1-\tau)^{\frac{\wp_2}{k}-1} E_{k,z_1,z_2}\left(\frac{-z^k}{k\tau(1-\tau)}\right) d\tau, \tag{2}$$

where $k > 0,\ \min\{\Re(\wp_1),\ \Re(\wp_2)\} > 0,\ z \geq 0,\ \Re(z_1) > 0,\ \Re(z_2) > 0$ and the k-Mittag-Leffler function is defined as follows:

$$E_{k,z_1,z_2}(\chi) = \sum_{i=0}^{\infty} \frac{\chi^i}{\Gamma_k(iz_1 + z_2)},\quad k > 0,\ \Re(z_1) > 0,\ \Re(z_2) > 0,\ \chi \in \mathbb{C}. \tag{3}$$

If we take $z_1 = z_2 = 1$, Eq. (3) reduces to the k-exponential function $E_k(\chi)$. For $k = 1$ in Eq. (3), the k-Mittag-Leffler function reduces to the usual two parameters Mittag-Leffler function.

$$E_{z_1,z_2}(\chi) = \sum_{i=0}^{\infty} \frac{\chi^i}{\Gamma(iz_1 + z_2)},\quad \Re(z_1) > 0,\ \Re(z_2) > 0,\ \chi \in \mathbb{C}. \tag{4}$$

This extension allow hypergeometric functions to model a wide range of applied mathematics and engineering problems. The extended hypergeometric function introduce extra parameters that allow greater control over the function's behavior, making them more versatile for several applications in physics, financial modeling, and other fields. In this manuscript, we evaluate the Saigo k-fractional integral and derivative operators in relation to the newly extended k-hypergeometric function.

The remainder of the paper is structured as follows: In Section 2, we introduce the fundamental definitions of k-beta and gamma functions. Section 3 presents the generalized k-Saigo fractional approach. Section 4 provides the main results of the present work. Finally, concluding remarks are presented in Section 5.

2. Fundamental Concepts

This section revisits established results and key definitions essential for the study.

Definition 1. The k-Gamma function, $\Gamma_k(\chi)$, is defined as follows [1, 4]:

$$\Gamma_k(\chi) = \frac{\lim\limits_{i\to\infty} i!k^i (ik)^{\frac{\chi}{k}-1}}{(\chi)_{i,\ k}},\quad k > 0,\ \chi \in \mathbb{C}, \tag{5}$$

where $(\chi)_{i,\ k}$ is the k-Pochhamer notation and is given by [4]:

$$(\chi)_{i,\ k} = \chi(\chi+k)(\chi+2k)\ldots(\chi+ik-k),\quad \chi \in \mathbb{C},\ k > 0,\ i \in \mathbb{N}. \tag{6}$$

For $\Re(\chi) > 0$ and $k > 0$, then $\Gamma_k(\chi)$ has the following integral form:

$$\Gamma_k(\chi) = k^{\frac{\chi}{k}-1}\Gamma\left(\frac{\chi}{k}\right) = \int_0^{\infty} y^{\chi-1} e^{-\frac{y^k}{k}} dy. \tag{7}$$

Note that $\Gamma_k(\chi + k) = \chi\Gamma_k(\chi)$.

Definition 2. The generalized k-beta function, $\mathscr{B}_k(\chi,\ \tau)$, is defined as follows [1, 4]:

$$\mathscr{B}_k(\chi,\ \tau) = \frac{1}{k}\int_0^1 y^{\frac{\chi}{k}-1}(1-y)^{\frac{\tau}{k}-1} dy,\quad \chi,\ \tau > 0. \tag{8}$$

The following are key properties of the k-beta function and the k-Pochhammer symbol [1].

(a) $\mathscr{B}_k(\chi,\ \tau)=\frac{1}{k}\mathscr{B}_k\left(\frac{\chi}{k},\ \frac{\tau}{k}\right)$,

(b) $\mathscr{B}_k(\chi,\ \tau)=\frac{\Gamma_k(\chi)\Gamma_k(\tau)}{\Gamma_k(\chi+\tau)}$,

(c) $(\chi)_{i,\ k}=\frac{\Gamma_k(\chi+ik)}{\Gamma_k(\chi)}$.

Gehlot et al. [6] introduced the generalized k-Wright function ${}_u\Psi_v^k(\chi)$ defined for $k\in\mathbb{R}^+;\ \chi,\ \upsilon_i,\ \nu_j\in\mathbb{C},\ a_i,\ b_j\in\mathbb{R}\ (a_i,\ b_j\neq 0)$ where $i=1,\ 2,\ \ldots u;\ j=1,\ 2,\ \ldots v$ and $(\upsilon_i+a_im),\ (\nu_j+b_jm)\in\mathbb{C}\backslash k\mathbb{Z}^-$:

$$ {}_u\Psi_v^k(\chi)={}_u\Psi_v^k\left[\begin{matrix}(\upsilon_i,\ a_i)_{1,\ u}\\ (\nu_i,\ b_i)_{1,\ v}\end{matrix}\ ;\ \chi\right]=\sum_{m=0}^{\infty}\frac{\prod_{i=1}^{u}\Gamma_k(\upsilon_i+a_im)}{\prod_{j=1}^{v}\Gamma_k(\nu_j+b_jm)}\frac{\chi^m}{m!}, \tag{9}$$

which satisfies the condition below:

$$\sum_{j=1}^{v}\frac{b_j}{k}-\sum_{i=1}^{u}\frac{a_i}{k}>-1.$$

The introduction of k-type fractional calculus is a very important development in the field of classical fractional calculus due to the fact that it has proven to be widely applicable in many fields of mathematical, physical and applied sciences.

3. Generalized k-Saigo Fractional Approach

Next, we examine key characteristics of the left and right-sided Saigo k-fractional calculus operators defined by Gupta and Parihar [1]. Let $\eta,\ \sigma,\ \mu\in\mathbb{C},\ \Re(\eta)>0,\ k>0,\ \chi\in\mathbb{R}^+$; the generalized fractional differentiation and integration operators associated with the Gauss k-type hypergeometric function are defined as follows:

$$\left(I_{0+,k}^{\eta,\sigma,\mu}u\right)(\chi)=\frac{\chi^{\frac{-\eta-\sigma}{k}}}{k\Gamma_k(\eta)}\int_0^{\chi}(\chi-\tau)^{\frac{\eta}{k}-1}\ {}_2\mathbb{F}_{1,k}\left((\eta+\sigma,\ k),\ (-\mu,k);\ (\eta,k);\left(1-\frac{\tau}{\chi}\right)\right)u(\tau)\,d\tau; \tag{10}$$

$$\left(I_{-,k}^{\eta,\sigma,\mu}u\right)(\chi)=\frac{1}{k\Gamma_k(\eta)}\int_{\chi}^{\infty}(\tau-\chi)^{\frac{\eta}{k}-1}\tau^{\frac{-\eta-\sigma}{k}}\ {}_2\mathbb{F}_{1,k}\left((\eta+\sigma,k),(-\mu,k);(\eta,k);\left(1-\frac{\chi}{\tau}\right)\right)u(\tau)\,d\tau, \tag{11}$$

where ${}_2\mathbb{F}_{1,\ k}((\eta,\ k),\ (\sigma,\ k);\ (\mu,\ k);\ \chi)$ is the k-Gauss hypergeometric function defined by [7] for $\chi\in\mathbb{C},\ |\chi|<1,\ \Re(\mu)>\Re(\sigma)>0$

$${}_2\mathbb{F}_{1,\ k}((\eta,\ k),\ (\sigma,\ k);\ (\mu,\ k);\ \chi)=\sum_{i=0}^{\infty}\frac{(\eta)_{i,k}(\sigma)_{i,k}\chi^i}{(\mu)_{i,k}\,i!}. \tag{12}$$

The corresponding forms of Saigo k-fractional differential operators are provided below [1]:

$$\begin{aligned}\left(\mathscr{D}_{0+,k}^{\eta,\ \sigma,\ \mu}u\right)(\chi) &= \left(\frac{d}{d\chi}\right)^i\left(I_{0+,k}^{-\eta+i,\ -\sigma-i,\ \eta+\mu-i}u\right)(\chi),\quad \Re(\eta)>0,\ k>0,\ i=[\Re(\eta)+1] \\ &= \left(\frac{d}{d\chi}\right)^i\frac{\chi^{\frac{\eta+\sigma}{k}}}{k\Gamma_k(-\eta+i)}\int_0^{\chi}(\chi-\tau)^{-\frac{\eta}{k}+i-1} \\ &\times\ {}_2\mathbb{F}_{1,\ k}\left((-\eta-\sigma,\ k),\ (-\mu-\eta+i,\ k);\ (-\eta+i,\ k);\ \left(1-\frac{\tau}{\chi}\right)\right)u(\tau)\,d\tau.\end{aligned} \tag{13}$$

$$\left(\mathscr{D}_{-,k}^{\eta,\ \sigma,\ \mu}u\right)(\chi) = \left(-\frac{d}{d\chi}\right)^i\left(I_{-,k}^{-\eta+i,\ -\sigma-i,\ \eta+\mu}u\right)(\chi),\quad \Re(\eta)>0,\ k>0,\ i=[\Re(\eta)+1] \tag{14}$$

$$= \left(-\frac{d}{d\chi}\right)^{i} \frac{1}{k\Gamma_k\left(-\eta+i\right)} \int\limits_{\chi}^{\infty} \left(\tau-\chi\right)^{-\frac{\eta+i}{k}-1} \tau^{\frac{\eta+\sigma}{k}}$$

$$\times \quad {}_2\mathbb{F}_{1,\ k}\left(\left(-\eta-\sigma,\ k\right),\ \left(-\mu-\eta,\ k\right);\ \left(-\eta+i,\ k\right);\ \left(1-\frac{\chi}{\tau}\right)\right) u\left(\tau\right) d\tau.$$

where $\chi > 0,\ \eta \in \mathbb{C},\ \Re\left(\eta\right) > 0,\ k > 0$ and $\left[\Re\left(\eta\right)\right]$ is the integer part of $\Re\left(\eta\right)$.
For $k = 1$, the k-Saigo fractional calculus simplifies to the classical Saigo fractional calculus [5]. When $\sigma = -\eta$, the k-Saigo fractional operators reduce to the k-Riemann-Liouville fractional operators as follow:

$$\left(I_{0+,k}^{\eta,\ -\eta,\ \mu} u\right)\left(\chi\right) = \left(I_{0+,k}^{\eta} u\right)\left(\chi\right) \quad \text{and} \quad \left(I_{-,k}^{\eta,\ -\eta,\ \mu} u\right)\left(\chi\right) = \left(I_{-,k}^{\eta} u\right)\left(\chi\right). \tag{15}$$

$$\left(\mathscr{D}_{0+,k}^{\eta,\ -\eta,\ \mu} u\right)\left(\chi\right) = \left(\mathscr{D}_{0+,k}^{\eta} u\right)\left(\chi\right) \quad \text{and} \quad \left(\mathscr{D}_{-,k}^{\eta,\ -\eta,\ \mu} u\right)\left(\chi\right) = \left(\mathscr{D}_{-,k}^{\eta} u\right)\left(\chi\right). \tag{16}$$

Now, we consider the following basic results for our study.

4. Main Results

Next, we evaluate the Saigo k-fractional differentiation and integration related to the extended k-hypergeometric function (1), which are expressed in terms of the k-Wright function as shown in Eq. (9).

4.1. Saigo k-Fractional Integration of Extended k-Hypergeometric Function

Lemma 1. [1] Let $\eta,\ \sigma,\ \mu,\ \zeta \in C,\ \Re\left(\eta\right) > 0,\ k \in \mathbb{R}^{+}$

(a)If $\Re\left(\zeta\right) > \max\left[0,\ \Re\left(\sigma-\mu\right)\right]$, then

$$\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu} \tau^{\frac{\zeta}{k}-1}\right)\left(\chi\right) = \sum_{i=0}^{\infty} k^{i} \frac{\Gamma_k\left(\zeta\right)\Gamma_k\left(\zeta-\sigma+\mu\right)}{\Gamma_k\left(\zeta-\sigma\right)\Gamma_k\left(\zeta+\eta+\mu\right)} \chi^{\frac{\zeta-\sigma}{k}-1}. \tag{17}$$

(b)If $\Re\left(\zeta\right) > \max\left[\Re\left(-\sigma\right),\ \Re\left(-\mu\right)\right]$, then

$$\left(I_{-,\ k}^{\eta,\ \sigma,\ \mu} \tau^{-\frac{\zeta}{k}}\right)\left(\chi\right) = \sum_{i=0}^{\infty} k^{i} \frac{\Gamma_k\left(\zeta+\sigma\right)\Gamma_k\left(\zeta+\mu\right)}{\Gamma_k\left(\zeta\right)\Gamma_k\left(\zeta+\eta+\sigma+\mu\right)} \chi^{\frac{-\zeta-\sigma}{k}}. \tag{18}$$

Theorem 1. *Let $\eta,\ \sigma,\ \mu,\ \zeta\ \in\ \mathbb{C},\ c\ \in\ \mathbb{R}$ and $k > 0,\ \Re(\eta) > 0,\ \Re(\zeta+\mu-\sigma) > 0,\ \Re(\wp_3) > \Re(\wp_2) > s,\ \Re(z_1) > 0,\ \Re(z_2) > 0,\ |\chi| < \frac{1}{k}$. The left-sided Saigo k-fractional integration of the extended k-hypergeometric function is defined as follows:*

$$\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu} \tau^{\frac{\zeta}{k}-1} \mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{\left(\wp_1,\ k\right),\ \left(\wp_2,\ k\right);\ \left(\wp_3,\ k\right);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)\left(\chi\right) = \frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k\left(\wp_2-sk\right)\Gamma_k\left(\wp_3\right)}{\Gamma_k\left(\wp_1\right)\Gamma_k\left(\wp_2\right)\Gamma_k\left(\wp_3-\wp_2+sk\right)}$$

$$\times {}_5\Psi_5^{k}\left(\begin{array}{c} \left(\wp_3-\wp_2+sk,\ -k\right),\ \left(\wp_1,\ k\right),\ \left(\wp_2,\ k\right),\ \left(\zeta,\ \varsigma\right),\ \left(\zeta-\sigma+\mu,\ \varsigma\right) \\ \left(z_2,\ z_1\right),\ \left(\wp_3,\ -k\right),\ \left(\wp_2-sk,\ k\right),\ \left(\zeta-\sigma,\ \varsigma\right),\ \left(\zeta+\eta+\mu,\ \varsigma\right) \end{array};\ -z^{k}c\chi^{\frac{\varsigma}{k}}\right). \tag{19}$$

Proof. By applying Eq. (1) and Eq. (17) to the left side of Eq. (19), we obtain:

$$\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)(\chi)$$

$$=\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{\left(c\tau^{\frac{\varsigma}{k}}\right)^i}{i!}\right)(\chi)$$

$$=\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{c^i}{i!}\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta+\varsigma i}{k}-1}\right)(\chi)$$

$$=\chi^{\frac{\zeta-\sigma}{k}-1}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{1}{i!}$$

$$\times\frac{\Gamma_k(\zeta+\varsigma i)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}\left(ck\chi^{\frac{\varsigma}{k}}\right)^i$$

which upon the properties of k-beta function and definition of extended k-beta function, yields:

$$=\chi^{\frac{\zeta-\sigma}{k}-1}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{1}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{\Gamma_k(\zeta+\varsigma i)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}$$

$$\times\left(\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk+ik}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk}{k}-1}\right)E_{k,\ z_1,\ z_2}\left(\frac{-z^k}{k\tau(1-\tau)}d\tau\right)\frac{\left(ck\chi^{\frac{\varsigma}{k}}\right)^i}{i!}$$

The definition of the Mittag-Leffler function results in the following form:

$$=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk+ik}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk}{k}-1}\sum_{i=0}^{\infty}\frac{\left(\frac{-z^k}{k\tau(1-\tau)}\right)^i}{\Gamma_k(iz_1+z_2)}d\tau$$

$$\times\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\Gamma_k(\zeta+\varsigma n)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}\frac{\left(ck\chi^{\frac{\varsigma}{k}}\right)^i}{i!}$$

$$=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{\left(-z^k\right)^i}{k^i\Gamma_k(iz_1+z_2)}\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk-ik}{k}-1}d\tau$$

$$\times\frac{\Gamma_k(\wp_1+ik)}{\Gamma_k(\wp_1)}\frac{\Gamma_k(\wp_2+ik)}{\Gamma_k(\wp_2)}\frac{\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_2-sk+ik)}\frac{\Gamma_k(\zeta+\varsigma i)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}\frac{\left(ck\chi^{\frac{\varsigma}{k}}\right)^i}{i!}$$

$$=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}(-1)^i\frac{z^{ik}}{k^i\Gamma_k(iz_1+z_2)}\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk-ik)$$

$$\times\frac{\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)}\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\zeta+\varsigma i)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\wp_2-sk+ik)\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}\frac{\left(ck\chi^{\frac{\varsigma}{k}}\right)^i}{i!}$$

and with the use of relationship between k-beta and k-gamma function, we have:

$$=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k(\wp_2-sk)\Gamma_k(\wp_3)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}(-1)^i\frac{z^{ik}}{\Gamma_k(iz_1+z_2)}\frac{\Gamma_k(\wp_3-\wp_2+sk-ik)}{\Gamma_k(\wp_3-ik)}$$

$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\zeta+\varsigma i)\Gamma_k(\zeta+\varsigma i-\sigma+\mu)}{\Gamma_k(\wp_2-sk+ik)\Gamma_k(\zeta+\varsigma i-\sigma)\Gamma_k(\zeta+\varsigma i+\eta+\mu)}\frac{\left(c\chi^{\frac{\varsigma}{k}}\right)^i}{i!}$$

After simplification and applying the definition of the k-Wright function, we arrive at the desired objective: □

Theorem 2. *Let $\eta,\ \sigma,\ \mu,\ \zeta \in \mathbb{C},\ c \in \mathbb{R}$ and $k > 0,\ \Re(\eta) > 0,\ \Re(\zeta+\mu-\sigma) > 0,\ \Re(\wp_3) > \Re(\wp_2) > s,\ \Re(z_1) > 0,\ \Re(z_2) > 0,\ |\chi| < \frac{1}{k}$. The right-sided Saigo k-fractional integration of the extended k-hypergeometric function is defined as follows:*

$$I_{-,\ k}^{\eta,\ \sigma,\ \mu}\left(\tau^{\frac{-\eta-\zeta}{k}}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi) = \chi^{\frac{-\eta-\zeta-\sigma}{k}}\frac{\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$

$$\times{}_5\Psi_5^k\left(\begin{array}{c}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\eta+\zeta+\sigma,\ \varsigma),\ (\eta+\zeta+\mu,\ \varsigma)\\(z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\eta+\zeta,\ \varsigma),\ (2\eta+\zeta+\sigma+\mu,\ \varsigma)\end{array};\ -z^k c\chi^{-\frac{\varsigma}{k}}\right). \tag{20}$$

Proof. By applying Eq. (1) and Eq. (18) to the left side of Eq. (20), we have:

$$I_{-,\ k}^{\eta,\ \sigma,\ \mu}\left(\tau^{\frac{-\eta-\zeta}{k}}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi)$$

$$= \left(I_{-,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{-\eta-\zeta}{k}}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{\left(c\tau^{-\frac{\varsigma}{k}}\right)^i}{i!}\right)(\chi)$$

$$= \sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{c^i}{i!}\left(I_{-,\ k}^{\eta,\ \sigma,\ \mu}\tau^{-\frac{\varsigma i+\eta+\zeta}{k}}\right)(\chi)$$

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{1}{i!}$$

$$\times\frac{\Gamma_k(\eta+\zeta+\sigma+\varsigma i)\Gamma_k(\eta+\zeta+\mu+\varsigma i)}{\Gamma_k(\eta+\zeta+\varsigma i)\Gamma_k(2\eta+\zeta+\sigma+\mu+\varsigma i)}\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i$$

The properties of k-beta function and definition of extended k-beta function result in the following form:

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}}\frac{\Gamma_k(\wp_3)}{\Gamma_k(\wp_2-sk)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk+ik}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk}{k}-1}E_{k,\ z_1,\ z_2}\left(\frac{-z^k}{k\tau(1-\tau)}\right)d\tau$$

$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\wp_2-sk)\Gamma_k(\eta+\zeta+\sigma+\varsigma i)\Gamma_k(\eta+\zeta+\mu+\varsigma i)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2-sk+ik)\Gamma_k(\eta+\zeta+\varsigma i)\Gamma_k(2\eta+\zeta+\sigma+\mu+\varsigma i)}\frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

The definition of the Mittag-Leffler function results in the following form:

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}}\frac{\Gamma_k(\wp_3)}{\Gamma_k(\wp_2-sk)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk+ik}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk}{k}-1}\sum_{i=0}^{\infty}\frac{\left(\frac{-z^k}{k\tau(1-\tau)}\right)^i}{\Gamma_k(iz_1+z_2)}d\tau$$

$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\wp_2-sk)\Gamma_k(\eta+\zeta+\sigma+\varsigma i)\Gamma_k(\eta+\zeta+\mu+\varsigma i)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2-sk+ik)\Gamma_k(\eta+\zeta+\varsigma i)\Gamma_k(2\eta+\zeta+\sigma+\mu+\varsigma i)}\frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}}\frac{\Gamma_k(\wp_3)}{\Gamma_k(\wp_2-sk)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{1}{k}\sum_{i=0}^{\infty}(-1)^i\frac{z^{ik}}{k^i\Gamma_k(iz_1+z_2)}\int_0^1\tau^{\frac{\wp_2-sk}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk-ik}{k}-1}d\tau$$

$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\wp_2-sk)\Gamma_k(\eta+\zeta+\sigma+\varsigma i)\Gamma_k(\eta+\zeta+\mu+\varsigma i)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2-sk+ik)\Gamma_k(\eta+\zeta+\varsigma i)\Gamma_k(2\eta+\zeta+\sigma+\mu+\varsigma i)}\frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}} \frac{\Gamma_k(\wp_3)}{\Gamma_k(\wp_2 - sk)\Gamma_k(\wp_3 - \wp_2 + sk)} \sum_{i=0}^{\infty}\sum_{i=0}^{\infty} (-1)^i \frac{z^{ik}}{k^i \Gamma_k(iz_1 + z_2)} \mathscr{B}_k(\wp_2 - sk, \ \wp_3 - \wp_2 + sk - ik)$$

$$\times \frac{\Gamma_k(\wp_1 + ik)\Gamma_k(\wp_2 + ik)\Gamma_k(\wp_2 - sk)\Gamma_k(\eta + \zeta + \sigma + \varsigma i)\Gamma_k(\eta + \zeta + \mu + \varsigma i)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2 - sk + ik)\Gamma_k(\eta + \zeta + \varsigma i)\Gamma_k(2\eta + \zeta + \sigma + \mu + \varsigma i)} \frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

The relationship of k-gamma and k-beta functions gives:

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}} \frac{\Gamma_k(\wp_3)}{\Gamma_k(\wp_3 - \wp_2 + sk)} \sum_{i=0}^{\infty}\sum_{i=0}^{\infty} (-1)^i \frac{z^{ik}}{k^i \Gamma_k(iz_1 + z_2)} \frac{\Gamma_k(\wp_3 - \wp_2 + sk - ik)}{\Gamma_k(\wp_3 - ik)}$$

$$\times \frac{\Gamma_k(\wp_1 + ik)\Gamma_k(\wp_2 + ik)\Gamma_k(\wp_2 - sk)\Gamma_k(\eta + \zeta + \sigma + \varsigma i)\Gamma_k(\eta + \zeta + \mu + \varsigma i)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2 - sk + ik)\Gamma_k(\eta + \zeta + \varsigma i)\Gamma_k(2\eta + \zeta + \sigma + \mu + \varsigma i)} \frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

Using properties of double summation with the same indices, we have:

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}} \frac{\Gamma_k(\wp_3)\Gamma_k(\wp_2 - sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3 - \wp_2 + sk)} \sum_{i=0}^{\infty} (-1)^i \frac{z^{ik}}{k^i \Gamma_k(iz_1 + z_2)} \frac{\Gamma_k(\wp_3 - \wp_2 + sk - ik)}{\Gamma_k(\wp_3 - ik)}$$

$$\times \frac{\Gamma_k(\wp_1 + ik)\Gamma_k(\wp_2 + ik)\Gamma_k(\eta + \zeta + \sigma + \varsigma i)\Gamma_k(\eta + \zeta + \mu + \varsigma i)}{\Gamma_k(\wp_2 - sk + ik)\Gamma_k(\eta + \zeta + \varsigma i)\Gamma_k(2\eta + \zeta + \sigma + \mu + \varsigma i)} \frac{\left(ck\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

Further simplification gives:

$$= \chi^{\frac{-\eta-\zeta-\sigma}{k}} \frac{\Gamma_k(\wp_3)\Gamma_k(\wp_2 - sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3 - \wp_2 + sk)} \sum_{i=0}^{\infty} \frac{\Gamma_k(\wp_3 - \wp_2 + sk - ik)}{\Gamma_k(iz_1 + z_2)\Gamma_k(\wp_3 - ik)}$$

$$\times \frac{\Gamma_k(\wp_1 + ik)\Gamma_k(\wp_2 + ik)\Gamma_k(\eta + \zeta + \sigma + \varsigma i)\Gamma_k(\eta + \zeta + \mu + \varsigma i)}{\Gamma_k(\wp_2 - sk + ik)\Gamma_k(\eta + \zeta + \varsigma i)\Gamma_k(2\eta + \zeta + \sigma + \mu + \varsigma i)} \frac{\left(-z^k c\chi^{-\frac{\varsigma}{k}}\right)^i}{i!}$$

After simplification and applying the definition of the k-Wright function, we arrive at the desired objective: □

Special cases:

1.For $\sigma = -\eta$, the left-sided Riemann-Liouville k-fractional integral of the extended k-hypergeometric function is defined as follows:

$$\left(I_{0+,\ k}^{\eta,\ -\eta,\ \mu} \tau^{\frac{\zeta}{k}-1} \mathbb{F}_{k,\ z}^{(z_1,\ z_2)} \left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)(\chi) = \frac{\chi^{\frac{\zeta+\eta}{k}-1}\Gamma_k(\wp_2 - sk)\Gamma_k(\wp_3)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3 - \wp_2 + sk)}$$

$$\times {}_5\Psi_5^k \left(\begin{matrix} (\wp_3 - \wp_2 + sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma),\ (\zeta + \eta + \mu,\ \varsigma) \\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2 - sk,\ k),\ (\zeta + \eta,\ \varsigma),\ (\zeta + \eta + \mu,\ \varsigma) \end{matrix} ;\ -z^k c\chi^{\frac{\varsigma}{k}} \right). \tag{21}$$

2.For $\sigma = -\eta$, the right-sided Riemann-Liouville k-fractional integral of the extended k-hypergeometric function is defined as follows:

$$I_{-,\ k}^{\eta,\ -\eta,\ \mu} \left(\tau^{\frac{-\eta-\zeta}{k}} \mathbb{F}_{k,\ z}^{(z_1,\ z_2)} \left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi) = \chi^{-\frac{\zeta}{k}} \frac{\Gamma_k(\wp_3)\Gamma_k(\wp_2 - sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3 - \wp_2 + sk)}$$

$$\times {}_5\Psi_5^k \left(\begin{matrix} (\wp_3 - \wp_2 + sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma),\ (\eta + \zeta + \mu,\ \varsigma) \\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2 - sk,\ k),\ (\eta + \zeta,\ \varsigma),\ (\eta + \zeta + \mu,\ \varsigma) \end{matrix} ;\ -z^k c\chi^{-\frac{\varsigma}{k}} \right). \tag{22}$$

4.2. Saigo k-Fractional Differentiation of Extended k-Hypergeometric Function

Lemma 2. [1] Let $\eta,\ \sigma,\ \mu,\ \zeta\in\mathbb{C}$ and $i=\lceil\Re(\eta)\rceil+1,\ k\in\mathbb{R}^{+}(0,\ \infty)$.

(a)If $\Re(\zeta)>\max[0,\ \Re(-\eta-\sigma-\mu)]$, then

$$\left(\mathscr{D}_{0+,k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\right)(\chi)=\sum_{i=0}^{\infty}\frac{\Gamma_k(\zeta)\,\Gamma_k(\zeta+\sigma+\mu+\eta)}{\Gamma_k(\zeta+\mu)\,\Gamma_k(\zeta+\sigma+i-ik)}\chi^{\frac{\zeta+\sigma+i}{k}-i-1}. \tag{23}$$

(b)If $\Re(\zeta)>\max[\Re(-\eta-\mu),\ \Re(\sigma-ik+i)]$, then

$$\left(\mathscr{D}_{-,k}^{\eta,\ \sigma,\ \mu}\tau^{-\frac{\zeta}{k}}\right)(\chi)=\sum_{i=0}^{\infty}\frac{\Gamma_k(\zeta-\sigma-i+ik)\,\Gamma_k(\zeta+\mu+\eta)}{\Gamma_k(\zeta)\,\Gamma_k(\zeta-\sigma+\mu)}\chi^{\frac{-\zeta+\sigma+i}{k}-i}. \tag{24}$$

Theorem 3. *Let $\eta,\ \sigma,\ \mu,\ \zeta\ \in\ \mathbb{C},\ c\ \in\ \mathbb{R}$ and $k\ >\ 0,\ \Re(\eta)\ >\ 0,\ \Re(\zeta+\mu+\sigma)\ >\ 0,\ \Re(\wp_3)\ >\ \Re(\wp_2)\ >\ s,\ \Re(z_1)\ >\ 0,\ \Re(z_2)\ >\ 0,\ |\chi|\ <\ \frac{1}{k}$. The left-sided Saigo k-fractional derivative of the extended k-hypergeometric function is defined as follows*

$$\left(\mathscr{D}_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)(\chi)=\frac{\chi^{\frac{\zeta+\sigma}{k}-1}\Gamma_k(\wp_3)\,\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\,\Gamma_k(\wp_2)\,\Gamma_k(\wp_3-\wp_2+sk)}$$
$$ {}_5\Psi_5^k\left(\begin{array}{c}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma),\ (\zeta+\sigma+\mu+\eta,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\zeta+\mu,\ \varsigma),\ (\zeta+\sigma,\ \varsigma+1-k)\end{array};\ -\frac{z^k}{k}c\chi^{\frac{\varsigma+1}{k}-1}\right). \tag{25}$$

Proof. By applying Eq. (1) and Eq. (23) to the left side of Eq. (25), we have:

$$\left(\mathscr{D}_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)(\chi)$$
$$=\left(\mathscr{D}_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta}{k}-1}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{\left(c\tau^{\frac{\varsigma}{k}}\right)^i}{i!}\right)(\chi)$$
$$=\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{c^i}{i!}\left(\mathscr{D}_{0+,\ k}^{\eta,\ \sigma,\ \mu}\tau^{\frac{\zeta+\varsigma i}{k}-1}\right)(\chi)$$
$$=\frac{\chi^{\frac{\zeta+\sigma}{k}-1}\Gamma_k(\wp_3)}{\Gamma_k(\wp_2-sk)\,\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\sum_{i=0}^{\infty}(-1)^i\frac{z^{ik}}{k^i\Gamma_k(iz_1+z_2)}\frac{1}{k}\int_0^1\tau^{\frac{\wp_2-sk}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk-ik}{k}-1}d\tau$$
$$\times\frac{\Gamma_k(\wp_1+ik)\,\Gamma_k(\wp_2+ik)\,\Gamma_k(\wp_2-sk)\,\Gamma_k(\zeta+\varsigma i)\,\Gamma_k(\zeta+\varsigma i+\sigma+\mu+\eta)}{\Gamma_k(\wp_1)\,\Gamma_k(\wp_2)\,\Gamma_k(\wp_2-sk+ik)\,\Gamma_k(\zeta+\varsigma i+\mu)\,\Gamma_k(\zeta+\varsigma i+\sigma+i-ik)}\frac{\left(c\chi^{\frac{\varsigma+1}{k}-1}\right)^i}{i!}$$

Applying the same simplification process used in the preceding theorems, we obtain:

$$=\frac{\chi^{\frac{\zeta+\sigma}{k}-1}\Gamma_k(\wp_3)\,\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\,\Gamma_k(\wp_2)\,\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{\Gamma_k(\wp_3-\wp_2+sk-ik)}{\Gamma_k(iz_1+z_2)\,\Gamma_k(\wp_3-ik)}$$
$$\times\frac{\Gamma_k(\wp_1+ik)\,\Gamma_k(\wp_2+ik)\,\Gamma_k(\zeta+\varsigma i)\,\Gamma_k(\zeta+\varsigma i+\sigma+\mu+\eta)}{\Gamma_k(\wp_2-sk+ik)\,\Gamma_k(\zeta+\varsigma i+\mu)\,\Gamma_k(\zeta+\varsigma i+\sigma+i-ik)}\frac{\left(-\frac{z^k}{k}c\chi^{\frac{\varsigma+1}{k}-1}\right)^i}{i!}.$$

After simplification and applying the definition of the k-Wright function, we arrive at the desired objective:
□

Theorem 4. *Let $\eta,\ \sigma,\ \mu,\ \zeta \in \mathbb{C},\ c \in \mathbb{R}$ and $k > 0,\ \Re(\eta) > 0,\ \Re(\zeta+\mu+\sigma) > 0,\ \Re(\wp_3) > \Re(\wp_2) > s,\ \Re(z_1) > 0,\ \Re(z_2) > 0,\ |\chi| < \frac{1}{k}$. The right-sided Saigo k-fractional derivative of the extended k-hypergeometric function is defined as follows*

$$\left(\mathscr{D}^{\eta,\ \sigma,\ \mu}_{-,\ k}\tau^{\frac{\eta-\zeta}{k}}\mathbb{F}^{(z_1,\ z_2)}_{k,\ z}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi) = \frac{\chi^{\frac{\eta+\sigma-\zeta}{k}}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$
$$ {}_5\Psi^k_5\left(\begin{matrix}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta-\eta-\sigma,\ \varsigma-1+k),\ (\zeta+\mu,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\zeta-\eta,\ \varsigma),\ (\zeta-\eta-\sigma+\mu,\ \varsigma)\end{matrix};\ -\frac{z}{k}^k c\chi^{\frac{-\varsigma+1}{k}-1}\right). \tag{26}$$

Proof. Applying Eq. (1) and Eq. (24) in the left side of Eq. (26), we have:

$$\left(\mathscr{D}^{\eta,\ \sigma,\ \mu}_{-,\ k}\tau^{\frac{\eta-\zeta}{k}}\mathbb{F}^{(z_1,\ z_2)}_{k,\ z}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi)$$
$$= \left(\mathscr{D}^{\eta,\ \sigma,\ \mu}_{-,\ k}\tau^{\frac{\eta-\zeta}{k}}\sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}^{(z_1,\ z_2)}_{k,\ z}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{\left(c\tau^{-\frac{\varsigma}{k}}\right)^i}{i!}\right)(\chi)$$
$$= \sum_{i=0}^{\infty}\frac{(\wp_1)_{i,\ k}(\wp_2)_{i,\ k}}{(\wp_2-sk)_{i,\ k}}\frac{\mathscr{B}^{(z_1,\ z_2)}_{k,\ z}(\wp_2-sk+ik,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{c^i}{i!}\left(\mathscr{D}^{\eta,\ \sigma,\ \mu}_{-,\ k}\tau^{-\frac{\varsigma i+\zeta-\eta}{k}}\right)(\chi)$$
$$= \frac{\chi^{\frac{\eta+\sigma-\zeta}{k}}\Gamma_k(\wp_3)}{\Gamma_k(\wp_2-sk)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\sum_{i=0}^{\infty}(-1)^i\frac{z^{ik}}{k^i\Gamma_k(iz_1+z_2)}\int_0^1\tau^{\frac{\wp_2-sk}{k}-1}(1-\tau)^{\frac{\wp_3-\wp_2+sk-ik}{k}-1}d\tau$$
$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\wp_2-sk)\Gamma_k(\zeta-\eta-\sigma+(\varsigma-1+k)i)\Gamma_k(\varsigma i+\zeta+\mu)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_2-sk+ik)\Gamma_k(\varsigma i+\zeta-\eta)\Gamma_k(\varsigma i+\zeta-\eta-\sigma+\mu)}\frac{\left(c\chi^{\frac{-\varsigma+1}{k}-1}\right)^i}{i!}$$

Further simplification gives:

$$= \frac{\chi^{\frac{\eta+\sigma-\zeta}{k}}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{i=0}^{\infty}\frac{\Gamma_k(\wp_3-\wp_2+sk-ik)}{\Gamma_k(iz_1+z_2)\Gamma_k(\wp_3-ik)}$$
$$\times\frac{\Gamma_k(\wp_1+ik)\Gamma_k(\wp_2+ik)\Gamma_k(\zeta-\eta-\sigma+(\varsigma-1+k)i)\Gamma_k(\varsigma i+\zeta+\mu)}{\Gamma_k(\wp_2-sk+ik)\Gamma_k(\varsigma i+\zeta-\eta)\Gamma_k(\varsigma i+\zeta-\eta-\sigma+\mu)}\frac{\left(-\frac{z}{k}^k c\chi^{\frac{-\varsigma+1}{k}-1}\right)^i}{i!}.$$

After simplification and applying the definition of the k-Wright function, we arrive at the desired objective: □

Special cases:

1.For $\sigma = -\eta$, the left-sided Riemann-Liouville k-fractional derivative of the extended k-hypergeometric function is defined as follows:

$$\left(\mathscr{D}^{\eta,\ -\eta,\ \mu}_{0+,\ k}\tau^{\frac{\zeta}{k}-1}\mathbb{F}^{(z_1,\ z_2)}_{k,\ z}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{\frac{\varsigma}{k}}\right)\right\}\right)(\chi) = \frac{\chi^{\frac{\zeta-\eta}{k}-1}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$
$$ {}_5\Psi^k_5\left(\begin{matrix}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma),\ (\zeta+\mu,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\zeta+\mu,\ \varsigma),\ (\zeta-\eta,\ \varsigma+1-k)\end{matrix};\ -\frac{z}{k}^k c\chi^{\frac{\varsigma+1}{k}-1}\right). \tag{27}$$

2.For $\sigma=-\eta$, the right-sided Riemann-Liouville k-fractional derivative of the extended k-hypergeometric function is defined as follows:

$$\left(\mathscr{D}_{-,\ k}^{\eta,\ -\eta,\ \mu}\tau^{\frac{\eta-\zeta}{k}}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ \left(c\tau^{-\frac{\varsigma}{k}}\right)\right\}\right)(\chi)=\frac{\chi^{-\frac{\zeta}{k}}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$

$$ {}_5\Psi_5^k\left(\begin{array}{c}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma-1+k),\ (\zeta+\mu,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\zeta-\eta,\ \varsigma),\ (\zeta+\mu,\ \varsigma)\end{array};\ -\frac{z^k}{k}c\chi^{\frac{-\varsigma+1}{k}-1}\right). \tag{28}$$

4.3. Image Formulae Associated with Integral Transforms

In this section, we present solutions related to those obtained in previous subsections, particularly involving the integral transform.

Definition 3. For $\chi>0,\ \tau>0$ and $k\in\mathbb{R}^+$, the k-beta function can be defined in the following form:

$$\mathscr{B}_k(u(t);\ \chi,\ \tau)=\frac{1}{k}\int_0^1 t^{\frac{\chi}{k}-1}(1-t)^{\frac{\tau}{k}-1}u(t)\,dt. \tag{29}$$

Theorem 5. *Let $\eta,\ \sigma,\ \mu,\ \zeta\in\mathbb{C},\ c\in\mathbb{R}$ and $k>0,\ \Re(\eta)>0,\ \Re(\zeta+\mu-\sigma)>0,\ \Re(\wp_3)>\Re(\wp_2)>s,\ \Re(z_1)>0,\ \Re(z_2)>0$. then the following fractional order integral holds true:*

$$B_k\left[\left(I_{0+,k}^{\eta,\sigma,\mu}t^{\frac{\zeta}{k}-1}\mathbb{F}_{k,z}^{(z_1,z_2)}\left\{(\wp_1,k),(\wp_2,k);(\wp_3,k);\left(yt^{\frac{\varsigma}{k}}\right)\right\}\right)(x);\chi,\tau\right]=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)\Gamma_k(\tau)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$

$$\times{}_6\Psi_6^k\left(\begin{array}{c}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\zeta,\ \varsigma),\ (\zeta-\sigma+\mu,\ \varsigma),\ (\chi,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\zeta-\sigma,\ \varsigma),\ (\zeta+\eta+\mu,\ \varsigma),\ (\chi+\tau,\ \varsigma)\end{array};\ -z^kx^{\frac{\varsigma}{k}}\right). \tag{30}$$

Proof. Let $\mathscr{L}$ be the left-hand side of Eq. (30) and using Eq. (29), we have

$$\mathscr{L}=\frac{1}{k}\int_0^1 y^{\frac{\chi}{k}-1}(1-y)^{\frac{\tau}{k}-1}\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu}t^{\frac{\zeta}{k}-1}\mathbb{F}_{k,\ z}^{(z_1,\ z_2)}\left\{(\wp_1,\ k),\ (\wp_2,\ k);\ (\wp_3,\ k);\ (yt)^{\frac{\varsigma}{k}}\right\}\right)(x)\,dy \tag{31}$$

Using Eq. (1) and changing the order of integration and summation, which is valid under the conditions of Theorem 1, yields

$$\mathscr{L}=\sum_{p=0}^{\infty}\frac{(\wp_1)_{p,\ k}(\wp_2)_{p,\ k}}{(\wp_2-sk)_{p,\ k}}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+pk,\ \wp_3-\wp_2+sk)}{\mathscr{B}_k(\wp_2-sk,\ \wp_3-\wp_2+sk)}\frac{1}{p!}\left(I_{0+,\ k}^{\eta,\ \sigma,\ \mu}t^{\frac{\zeta+p\varsigma}{k}-1}\right)(x)$$
$$\times\frac{1}{k}\int_0^1 y^{\frac{\chi+p\varsigma}{k}-1}(1-y)^{\frac{\tau}{k}-1}dy. \tag{32}$$

which upon Eq. (17) and Definition 2(a) in Eq. (32), we get

$$\mathscr{L}=\frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k(\wp_3)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{p=0}^{\infty}\frac{\mathscr{B}_{k,\ z}^{(z_1,\ z_2)}(\wp_2-sk+pk,\ \wp_3-\wp_2+sk)}{}\frac{\left(k\chi^{\frac{\varsigma}{k}}\right)^p}{p!}$$
$$\times\frac{\Gamma_k(\zeta+p\varsigma)\Gamma_k(\zeta+p\varsigma-\sigma+\mu)\Gamma_k(\wp_1+pk)\Gamma_k(\wp_2+pk)\Gamma_k(\chi+p\varsigma)\Gamma_k(\tau)}{\Gamma_k(\zeta+p\varsigma-\sigma)\Gamma_k(\zeta+p\varsigma+\eta+\mu)\Gamma_k(\wp_2-sk+pk)\Gamma_k(\chi+p\varsigma+\tau)} \tag{33}$$

The definition of extended k-beta and Mittag-Leffler functions result in the following form:

$$\mathscr{L} = \frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k(\wp_3)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{p=0}^{\infty}\frac{1}{k}\int_0^1 q^{\frac{\wp_2-sk}{k}-1}(1-q)^{\frac{\wp_3-\wp_2+sk-pk}{k}-1}dq\frac{\left(-z^k\chi^{\frac{\varsigma}{k}}\right)^p}{p!}$$
$$\times\frac{\Gamma_k(\zeta+p\varsigma)\Gamma_k(\zeta+p\varsigma-\sigma+\mu)\Gamma_k(\wp_1+pk)\Gamma_k(\wp_2+pk)\Gamma_k(\chi+p\varsigma)\Gamma_k(\tau)}{\Gamma_k(pz_1+z_2)\Gamma_k(\zeta+p\varsigma-\sigma)\Gamma_k(\zeta+p\varsigma+\eta+\mu)\Gamma_k(\wp_2-sk+pk)\Gamma_k(\chi+p\varsigma+\tau)} \tag{34}$$

Further simplification leads to the following form:

$$\mathscr{L} = \frac{\chi^{\frac{\zeta-\sigma}{k}-1}\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)\Gamma_k(\tau)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}\sum_{p=0}^{\infty}\frac{\Gamma_k(\wp_3-\wp_2+sk-pk)}{\Gamma_k(\wp_3-pk)}\frac{\left(-z^k\chi^{\frac{\varsigma}{k}}\right)^p}{p!}$$
$$\times\frac{\Gamma_k(\zeta+p\varsigma)\Gamma_k(\zeta+p\varsigma-\sigma+\mu)\Gamma_k(\wp_1+pk)\Gamma_k(\wp_2+pk)\Gamma_k(\chi+p\varsigma)}{\Gamma_k(pz_1+z_2)\Gamma_k(\zeta+p\varsigma-\sigma)\Gamma_k(\zeta+p\varsigma+\eta+\mu)\Gamma_k(\wp_2-sk+pk)\Gamma_k(\chi+p\varsigma+\tau)} \tag{35}$$

After simplification and applying the definition of the k-Wright function, we arrive at the desired objective: □

Theorem 6. *Let $\eta,\ \sigma,\ \mu,\ \zeta \in \mathbb{C},\ c \in \mathbb{R}$ and $k > 0,\ \Re(\eta) > 0,\ \Re(\zeta+\mu-\sigma) > 0,\ \Re(\wp_3) > \Re(\wp_2) > r,\ \Re(z_1) > 0,\ \Re(z_2) > 0$. then the following fractional order integral holds true:*

$$B_k\left[\left(I_{-,k}^{\eta,\sigma,\mu}t^{\frac{-\eta-\zeta}{k}}\mathbb{F}_{k,z}^{(z_1,z_2)}\left\{(\wp_1,k),(\wp_2,k);(\wp_3,k);\left(yt^{\frac{\varsigma}{k}}\right)\right\}\right)(x);\chi,\tau\right] = \chi^{\frac{-\eta-\zeta-\sigma}{k}}\frac{\Gamma_k(\tau)\Gamma_k(\wp_3)\Gamma_k(\wp_2-sk)}{\Gamma_k(\wp_1)\Gamma_k(\wp_2)\Gamma_k(\wp_3-\wp_2+sk)}$$
$$\times{}_6\Psi_6^k\left(\begin{matrix}(\wp_3-\wp_2+sk,\ -k),\ (\wp_1,\ k),\ (\wp_2,\ k),\ (\eta+\zeta+\sigma,\ \varsigma),\ (\eta+\zeta+\mu,\ \varsigma),\ (\chi,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -k),\ (\wp_2-sk,\ k),\ (\eta+\zeta,\ \varsigma),\ (2\eta+\zeta+\sigma+\mu,\ \varsigma),\ (\xi+\tau,\ \varsigma)\end{matrix};\ -z^k\chi^{-\frac{\varsigma}{k}}\right). \tag{36}$$

Proof. The proof follows a similar procedures to the one outlined in Theorem 5. □

4.4. Numerical Examples

If we take $\eta=\sigma=\mu=1,\ c=k=1$ and $\chi=\frac{1}{2}$ in Eq. (19) and Eq. (20), we obtain the following two formulas:

Corollary 1. Let $\Re(\zeta)>0,\ \Re(\wp_3)>\Re(\wp_2)>s,\ \Re(z_1)>0,\ \Re(z_2)>0,\ |\chi|<1$. The left-sided Saigo k-fractional integration of the extended k-hypergeometric function is defined as follows:

$$\left(I_{0+}^{1,\ 1,\ 1}\tau^{\zeta-1}\mathbb{F}_z^{(z_1,\ z_2)}\{(\wp_1,\ 1),\ (\wp_2,\ 1);\ (\wp_3,\ 1);\ \tau^{\varsigma}\}\right)(\chi) = \frac{2^{\sigma+1-\zeta}\Gamma(\wp_2-s)\Gamma(\wp_3)}{\Gamma(\wp_1)\Gamma(\wp_2)\Gamma(\wp_3-\wp_2+s)}$$
$$\times{}_5\Psi_5\left(\begin{matrix}(\wp_3-\wp_2+s,\ -1),\ (\wp_1,\ 1),\ (\wp_2,\ 1),\ (\zeta,\ \varsigma),\ (\zeta,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -1),\ (\wp_2-s,\ 1),\ (\zeta-1,\ \varsigma),\ (\zeta+2,\ \varsigma)\end{matrix};\ -\frac{z}{2^{\varsigma}}\right). \tag{37}$$

Corollary 2. Let $\Re(\zeta)>0,\ \Re(\wp_3)>\Re(\wp_2)>s,\ \Re(z_1)>0,\ \Re(z_2)>0,\ |\chi|<1$. The right-sided Saigo k-fractional integration of the extended k-hypergeometric function is defined as follows:

$$I_{-}^{1,\ 1,\ 1}\left(\tau^{-1-\zeta}\mathbb{F}_z^{(z_1,\ z_2)}\left\{(\wp_1,\ 1),\ (\wp_2,\ 1);\ (\wp_3,\ 1);\ \tau^{-\varsigma}\right\}\right)(\chi) = \chi^{-2-\zeta}\frac{\Gamma(\wp_3)\Gamma(\wp_2-s)}{\Gamma(\wp_1)\Gamma(\wp_2)\Gamma(\wp_3-\wp_2+s)}$$
$$\times{}_5\Psi_5\left(\begin{matrix}(\wp_3-\wp_2+sk,\ -1),\ (\wp_1,\ 1),\ (\wp_2,\ 1),\ (2+\zeta,\ \varsigma),\ (2+\zeta,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -1),\ (\wp_2-s,\ 1),\ (1+\zeta,\ \varsigma),\ (4+\zeta,\ \varsigma)\end{matrix};\ -\frac{z}{2^{-\varsigma}}\right). \tag{38}$$

If we take $\eta = 1.2,\ \sigma = 1.5,\ \mu = 1.8,\ c = k = 1$ and $\chi = \dfrac{1}{3}$ in Eq. (25) and Eq. (26), we obtain the following two formulas:

Corollary 3. Let $\Re(\zeta + \mu + \sigma) > 0,\ \Re(\wp_3) > \Re(\wp_2) > s,\ \Re(z_1) > 0,\ \Re(z_2) > 0,\ |\chi| < \frac{1}{k}$. The left-sided Saigo k-fractional derivative of the extended k-hypergeometric function is defined as follows

$$\left(\mathscr{D}_{0+}^{1.2,\ 1.5,\ 1.8}\tau^{\zeta-1}\mathbb{F}_z^{(z_1,\ z_2)}\left\{(\wp_1,\ 1),\ (\wp_2,\ 1);\ (\wp_3,\ 1);\ (c\tau^{\varsigma})\right\}\right)(\chi) = \frac{(1/3)^{\zeta+0.5}\,\Gamma(\wp_3)\,\Gamma(\wp_2 - s)}{\Gamma(\wp_1)\,\Gamma(\wp_2)\,\Gamma(\wp_3 - \wp_2 + s)}$$

$${}_5\Psi_5\left(\begin{array}{c}(\wp_3 - \wp_2 + s,\ -1),\ (\wp_1,\ 1),\ (\wp_2,\ 1),\ (\zeta,\ \varsigma),\ (\zeta + 4.5,\ \varsigma)\\ (z_2,\ z_1),\ (\wp_3,\ -1),\ (\wp_2 - s,\ 1),\ (\zeta + 1.8,\ \varsigma),\ (\zeta + 1.5,\ \varsigma)\end{array};\ -z\,(1/3)^{\varsigma}\right). \tag{39}$$

5. Conclusion

In conclusion, our study highlights the new extensions of k-fractional calculus formulae related to the k-type extended hypergeometric function. We have successfully evaluated the Saigo k-fractional derivatives and integrals. Additionally, we have considered several special cases involving k-Riemann-Liouville type fractional calculus operators.

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